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A 4-neutrino model with a Higgs triplet

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Abstract. We take as a starting point the Gelmini–Roncadelli model enlarged by a term with explicit lepton number violation in the Higgs potential and add a neutrino singlet field that is coupled via a scalar doublet to the usual leptons. This scenario allows us to take into account all three present indications in favor of neutrino oscillations provided by the solar, atmospheric, and LSND neutrino oscillation experiments. Furthermore, it suggests a model which reproduces naturally one of the two 4-neutrino mass spectra favored by the data. In this model, the solar neutrino problem is solved by large mixing MSW $\nu_e \to \nu_\tau$ transitions, and the atmospheric neutrino problem by transitions of ν_μ into a sterile neutrino.

1 Introduction

At present, there are three indications in favor of neutrino oscillations with three different scales of the corresponding neutrino mass-squared differences. Taking into account that in the LEP experiment, the number of light active neutrinos was determined to be three, it follows that at least one sterile neutrino is required to describe all present neutrino oscillation data (for reviews see, e.g., [1–4]). In the following we confine ourselves to the 4-neutrino case, which was discussed in many papers for a number of reasons (for an incomplete list, see [5]). From present experimental data, the nature of the 4-neutrino mass spectrum can be inferred [6–8], and information on the 4×4 unitary neutrino mixing matrix U, which is defined by

$$\nu_{\alpha L} = \sum_{j=1}^{4} U_{\alpha j} \nu_{jL} \quad \text{with} \quad \alpha = e, \mu, \tau, s, \tag{1}$$

can be obtained. In this relation, $\nu_{\alpha L}$ denotes the fields with definite flavors or types whereas ν_{jL} denotes the left-handed part of the neutrino mass eigenfields. The measurement of the up–down asymmetry of the atmospheric muon neutrino flux [9] allows one to draw definite conclusions on the types of possible neutrino mass spectra [11] for the whole range of the mass-squared difference $\Delta m^2_{\mathrm{LSND}}$ determined by the LSND experiment [10] and other short-baseline neutrino oscillation experiments. In this way, only two types of mass spectra with two pairs of close masses are allowed. These mass spectra can be characterized in the following way [6,11]:

$$\begin{aligned} &(\mathrm{A}) \quad \underbrace{\overbrace{m_1 < m_2}^{\mathrm{atm}} \ll \overbrace{m_3 < m_4}^{\mathrm{solar}}}_{\mathrm{LSND}}, \\ &(\mathrm{B}) \quad \underbrace{\overbrace{m_1 < m_2}^{\mathrm{solar}} \ll \overbrace{m_3 < m_4}^{\mathrm{atm}}}_{\mathrm{LSND}}. \end{aligned} \tag{2}$$

The task of accommodating a light sterile neutrino in an extension of the standard model poses serious problems to model builders. In particular, it seems difficult to reconcile the mass spectra (2) and the large mixing observed in atmospheric neutrino oscillations with the original see-saw mechanism [12]. However, models have been proposed exploiting the "singular see-saw mechanism" [13] which naturally achieve a large active–sterile neutrino mixing [14–16]. Since a large mixing angle $\nu_e \to \nu_s$ transition as a solution of the solar neutrino puzzle is not compatible with the solar neutrino data [17], the singular see-saw mechanism offers the possibility to explain the atmospheric neutrino anomaly by $\nu_\mu \to \nu_s$ oscillations.

A large active–sterile neutrino mixing seems to be excluded by big-bang nucleosynthesis if only fewer than 4 effective light neutrino degrees of freedom (N_ν) are allowed (see [18,7,19] and citations therein). However, the upper bound on N_ν depends in particular on the primordial deuterium abundance $(D/H)_P$ for which conflicting measurements exist. For the low value of $(D/H)_P$, the value of N_ν should be rather close to 3 [20], whereas a high ratio $(D/H)_P$ allows also values of N_ν around 4 [21]. In the following, we adopt the hypothesis that $N_\nu = 4$ is allowed.

In this paper, our starting point for constructing a 4-neutrino model is not the singular see-saw mechanism but an extension of the standard model in the scalar sector. Nevertheless, we will see that one can arrive at a scenario equivalent to the one obtained in [14]. The possible scalar multiplets extending the standard model are simply obtained by studying the representations of SU(2)×U(1) contained in all the fermionic bilinears that can be formed. Apart from the scalar doublet, there are only three possibilities: a triplet, a singlet with charge +1 and a singlet with charge +2 [22]. The basic and most prominent models founded upon these scalar multiplets are given by the models of Gelmini–Roncadelli (GR) [23], Zee [24], and

Babu [25], respectively, with Majorana neutrino masses at the tree-, one-, and two-loop level. Our discussion is based on the GR model. In its original version, [23] it possesses a spontaneously broken lepton number leading to a majoron and a light neutral scalar such that the Z^0 vector boson decay into these two scalars has a width of twice the decay width of $Z^0 \to \nu_\alpha \bar{\nu}_\alpha$, where ν_α denotes any of the three active neutrinos [26]. Since there is no room for such a decay according to the LEP measurements, we explicitly break the lepton number by a cubic term in the Higgs potential (see, e.g., [27]) in order to make the majoron heavy. The vacuum expectation value (VEV) of the neutral member of the Higgs triplet gives a Majorana mass matrix at the tree level for the active neutrinos. To incorporate a sterile neutrino singlet field ν_{sR} we couple it to the standard model lepton doublets via a Higgs doublet (for an analogous procedure in the framework of the Zee model, see [28]) and invoke a symmetry to forbid the mass term $\nu_{sR}^T C^{-1} \nu_{sR}$ in which C is the charge conjugation matrix. The main point of our scenario is to exploit the relation

$$|v_T| \ll v \,, \tag{3}$$

where v_T is the VEV of the Higgs triplet and v denotes the largest absolute value of the VEVs of the scalar doublets. A large triplet VEV would destroy the tree-level relation $M_W = M_Z \cos\theta_W$ between the W and Z^0 boson masses and the Weinberg angle and the precision measurements place a stringent bound on v_T [29]. With the two scales v and v_T, we will show that at this stage we have a model equivalent to the one described in [14]. Finally, we will introduce a discrete symmetry to achieve maximal ν_μ–ν_s mixing, to some extent without fine-tuning. In the final stage of our model, we will have three scalar doublets in addition to the Higgs triplet.

Other 4-neutrino models with Higgs triplets have been considered in [30].

The paper is organized as follows. In Sect. 2 we will present a thorough discussion of the GR model with explicit lepton number violation since this model is the basis of the remaining discussion in the paper. The sterile neutrino singlet will be introduced in Sect. 3. In this section, we will have large active–sterile mixing, but only the introduction of a horizontal symmetry in Sect. 4 will naturally restrict the large mixing to the muon neutrino. In Sect. 5, we will present the conclusions.

2 The Gelmini–Roncadelli model with explicit lepton number violation

In the GR model, the Yukawa Lagrangian in the lepton sector is given by [23]

$$\mathcal{L}_Y = \sum_{a,b} \left\{ - c_{ab}\, \bar{\ell}_{aR} \phi^\dagger L_b + \frac{1}{2} f_{ab} L_a^T C^{-1} \mathrm{I}\tau_2 \Delta L_b \right\} + \mathrm{h.c.} \,, \tag{4}$$

where $a, b = 1, 2, 3$ are the summation indices over the active neutrino degrees of freedom, and L_a, ℓ_{aR}, and ϕ denote the left-handed lepton doublets, the right-handed lepton singlets and the Higgs doublet, respectively. The Higgs triplet Δ is represented in the form of a 2×2 matrix. The coupling matrix for the Higgs triplet is symmetric, i.e., $f_{ab} = f_{ba}$. Under $U \in \mathrm{SU}(2)$, these multiplets transform as

$$L_a \to U L_a \,,\ \ell_{aR} \to \ell_{aR} \,,\ \phi \to U\phi \,,\ \Delta \to U \Delta U^\dagger \,. \tag{5}$$

Their U(1) transformation properties are determined by the hypercharges:

$$\begin{array}{c|cccc} & L_a & \ell_{aR} & \phi & \Delta \\ \hline Y & -1 & -2 & 1 & 2. \end{array} \tag{6}$$

Note that we are using the indices a, b instead of α, β (1). The two sets of indices are identical in a basis where the mass matrix of the charged leptons is diagonal. However, for reasons that will become clear later, we want to use the more general notation. The VEVs of the Higgs multiplets consistent with electric charge conservation are given by

$$\langle \phi \rangle_0 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix} \quad \text{and} \quad \langle \Delta \rangle_0 = \begin{pmatrix} 0 & 0 \\ v_T & 0 \end{pmatrix} . \tag{7}$$

The relation between the triplet $\mathbf{\Phi}$, the 2×2 matrix Δ, and the charged scalars contained in the triplet is found to be

$$\Delta = \mathbf{\Phi} \cdot \tau = \begin{pmatrix} H^+ & \sqrt{2} H^{++} \\ \sqrt{2} H^0 & -H^+ \end{pmatrix} \tag{8}$$

with

$$\mathbf{\Phi} = \begin{pmatrix} \frac{1}{\sqrt{2}} (H^0 + H^{++}) \\ \frac{-\mathrm{i}}{\sqrt{2}} (H^0 - H^{++}) \\ H^+ \end{pmatrix} . \tag{9}$$

The matrices τ_j $(j = 1, 2, 3)$ are the Pauli matrices. In (7) we have set $\langle H^0 \rangle_0 = v_T/\sqrt{2}$. The most general Higgs potential involving ϕ and Δ is written as

$$\begin{aligned} V(\phi, \Delta) = & \\ & a\, \phi^\dagger \phi + \frac{b}{2} \mathrm{Tr}\,(\Delta^\dagger \Delta) + c\, (\phi^\dagger \phi)^2 + \frac{d}{4} \left(\mathrm{Tr}\,(\Delta^\dagger \Delta) \right)^2 \\ & + \frac{e-h}{2} \phi^\dagger \phi\, \mathrm{Tr}\,(\Delta^\dagger \Delta) + \frac{f}{4} \mathrm{Tr}\,(\Delta^\dagger \Delta^\dagger)\, \mathrm{Tr}\,(\Delta \Delta) \\ & + h\, \phi^\dagger \Delta^\dagger \Delta \phi + \left(t\, \phi^\dagger \Delta \tilde{\phi} + \mathrm{h.c.} \right) , \end{aligned} \tag{10}$$

where $\tilde{\phi} \equiv \mathrm{i}\tau_2 \phi^*$. If the lepton number is assumed to be conserved one has to assign lepton number -2 to the Higgs triplet and 0 to the Higgs doublet [23] (see (4)). This lepton number is explicitly broken by the last term in the Higgs potential (10). Otherwise, this Higgs potential agrees with the one given in [23], with the same definition of the coupling constants. All parameters in the Higgs potential are real except t, which is complex in general.

When a global U(1) transformation is performed, v can always be chosen to be real and positive. Because of the t

term in the potential, we do not have a second global symmetry, the lepton number [23], to make v_T real. Furthermore, t can also be complex, and therefore we generally write $t = |t|\mathrm{e}^{\mathrm{i}\omega}$ and $v_T = w\mathrm{e}^{\mathrm{i}\gamma}$ with $w \equiv |v_T|$. We assume that the following orders of magnitude for the parameters in the potential hold:

$$a,\, b \sim v^2\,; \quad c,\, d,\, e,\, f,\, h \sim 1\,; \quad |t| \ll v\,. \tag{11}$$

The potential as a function of the VEVs is given by

$$V(\langle\phi\rangle_0, \langle\Delta\rangle_0) = \frac{1}{2}av^2 + \frac{1}{2}bw^2 + \frac{1}{4}cv^4 + \frac{1}{4}dw^4 + \frac{1}{4}(e-h)v^2w^2 + v^2w|t|\cos(\omega+\gamma)\,. \tag{12}$$

It has to be minimized as a function of the three parameters v, w, γ in order to obtain the relations between the VEVs and the parameters of the Higgs potential. Minimization with respect to γ, the phase of v_T, involves only the last term in (12) with the minimum at $\omega + \gamma = \pi$ or

$$v_T = -w\mathrm{e}^{-\mathrm{i}\omega} \quad \text{and} \quad v_T t = -w|t|\,. \tag{13}$$

With this relation, the other two minimum conditions are

$$a + cv^2 + \frac{e-h}{2}w^2 - 2|t|w = 0\,, \tag{14}$$

$$b + dw^2 + \frac{e-h}{2}v^2 - \frac{|t|}{w}v^2 = 0\,. \tag{15}$$

With the assumptions in (11) we find the approximate solution

$$v^2 \simeq -\frac{a}{c} \quad \text{and} \quad w \simeq |t|\,\frac{v^2}{b + (e-h)v^2/2}\,. \tag{16}$$

Thus we see that $w \sim |t|$, i.e., the triplet VEV is of the order of the parameter $|t|$ in the Higgs potential. The fine-tuning to get a small triplet VEV is therefore simply given by $|t| \ll v$, which should find an explanation in a more complete theory which has the GR model as a low-energy limit.[1] This situation is analogous to that in the standard model and the see-saw mechanism for light neutrino masses, where the large mass scale of the right-handed neutrino singlets is assumed to come, e.g., from grand unification.

Equations (4) and (7) give rise to the mass terms for the charged leptons and the neutrinos:

$$-\left(\bar{\ell}_R \mathcal{M}_\ell \ell_L + \text{h.c.}\right) \quad \text{with} \quad \mathcal{M}_\ell = \frac{v}{\sqrt{2}}\left(c_{ab}\right)\,, \tag{17}$$

$$\frac{1}{2}\nu_L^T C^{-1}\mathcal{M}_\nu \nu_L + \text{h.c.} \quad \text{with} \quad \mathcal{M}_\nu = v_T\left(f_{ab}\right)\,. \tag{18}$$

As mentioned earlier, if the cubic term in the potential (10) is absent, then there are two independent symmetries, the gauge group and the lepton number, which allow us to adopt the convention v and v_T as both real and positive. This means that in the Higgs sector CP cannot be broken. It could, of course, be violated explicitly by complex Yukawa couplings. In the presence of the cubic term, the situation is more complicated. We define a CP transformation

$$\phi \to \phi^*\,, \quad \Delta \to \rho\Delta^* \quad \text{with} \quad |\rho| = 1 \tag{19}$$

for the two scalar multiplets. Invariance of the Higgs potential under this CP transformation leads to the condition

$$t^* = \rho t \tag{20}$$

for the parameter t. Interpreted in another way, for any complex phase ω of t, the Higgs potential is invariant under the CP transformation (19) if we choose

$$\rho = \mathrm{e}^{-2\mathrm{i}\omega}\,. \tag{21}$$

Let us check that the VEVs are indeed invariant under the CP symmetry defined by (19) and (21). This is clear for $\langle\phi\rangle_0$ since v is real. Taking into account that the phase of v_T is given by (13) at the minimum of the potential and using (19) and (21), we find

$$\langle\Delta\rangle_0 = \begin{pmatrix} 0 & 0 \\ v_T & 0 \end{pmatrix} \xrightarrow{\text{CP}} \rho\langle\Delta\rangle_0^* = \rho\begin{pmatrix} 0 & 0 \\ v_T^* & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ v_T & 0 \end{pmatrix}\,. \tag{22}$$

Hence we see that the vacuum state is invariant under CP, regardless of the complex phase of t in the Higgs potential (10) [31], and thus CP cannot be spontaneously broken. Extending the CP transformation (19) by

$$\Psi(x^0, \mathbf{x}) \to -C\Psi^*(x^0, -\mathbf{x}) \tag{23}$$

for the fermionic multiplets and assuming that the vector bosons transform in the usual way, we obtain the conditions

$$c_{ab} = c_{ab}^*\,, \quad -\rho f_{ab} = f_{ab}^* \tag{24}$$

for CP invariance of the fermionic Lagrangian. Using the second relation in (24), we find with (21) that

$$f_{ab}^* = -\mathrm{e}^{-2\mathrm{i}\omega} f_{ab}\,. \tag{25}$$

If we define f'_{ab} by

$$f'_{ab} = \mathrm{i}\mathrm{e}^{-\mathrm{i}\omega} f_{ab}, \tag{26}$$

then (25) implies

$$f'_{ab} \in \mathbf{R} \quad \text{and} \quad v_T f_{ab} = \mathrm{i}w f'_{ab}\,. \tag{27}$$

In the following we will assume CP invariance for simplicity, though it is not essential for the construction of our model.

In the GR model the relation between the W and Z^0 masses is obtained as [23,29]

$$\frac{M_W^2}{M_Z^2\cos^2\theta_W} = \frac{1 + 2w^2/v^2}{1 + 4w^2/v^2}\,, \tag{28}$$

[1] Alternatively, one could use $b \gg v^2$ to get a small triplet VEV [27].

whereas in the standard model this ratio is 1. From precision data, in [29] the bound

$$\frac{w}{v} \lesssim 0.03 \tag{29}$$

was obtained at 95% CL. If there are several Higgs doublets with VEVs v_k, then v has to replaced by $(\sum_k |v_k|^2)^{1/2}$ in (29).

With the definitions

$$\phi^0 = \frac{1}{\sqrt{2}}(v + \phi_R + \mathrm{i}\phi_I)\,, \quad H^0 = \frac{1}{\sqrt{2}}\mathrm{e}^{\mathrm{i}\gamma}(w + H_R + \mathrm{i}H_I)\,, \tag{30}$$

where the scalar fields with the subscripts R and I are real fields, we write the couplings of the neutral scalars to the Z^0 boson as

$$\frac{\sqrt{g^2+g'^2}}{2}\, Z^\mu \,\{(\partial_\mu \phi_R)\phi_I - (\partial_\mu \phi_I)(\phi_R + v) + 2(\partial_\mu H_R)H_I - 2(\partial_\mu H_I)(H_R + w)\}\,. \tag{31}$$

The quantities g and g' are the gauge coupling constants of SU(2)×U(1). Note that the linear combination

$$2wH_I + v\phi_I \tag{32}$$

in (31) is proportional to the pseudo-Goldstone boson field associated with the Z^0.

CP invariance of the scalar sector under the transformation (19) and the decomposition (30) show that the R fields are CP-even and the I fields CP-odd. Therefore, the mass matrix of the four real scalar fields splits into two separate 2×2 matrices for the real and imaginary parts, i.e.,

$$\mathcal{L}_S^0 = -\frac{1}{2}(H_R, \phi_R)\mathcal{M}_R^2 \begin{pmatrix} H_R \\ \phi_R \end{pmatrix} - \frac{1}{2}(H_I, \phi_I)\mathcal{M}_I^2 \begin{pmatrix} H_I \\ \phi_I \end{pmatrix}. \tag{33}$$

Using the minimum conditions (14) and (15) we get

$$\mathcal{M}_R^2 = \begin{pmatrix} 2dw^2 + qv^2 & (e-h-2q)wv \\ (e-h-2q)wv & 2cv^2 \end{pmatrix}, \tag{34}$$

$$\mathcal{M}_I^2 = q \begin{pmatrix} v^2 & -2wv \\ -2wv & 4w^2 \end{pmatrix}, \tag{35}$$

where we have defined

$$q \equiv |t|/w\,, \tag{36}$$

which is a positive quantity of order one according to the assumptions (11). The eigenvalues of the matrices $\mathcal{M}_R^2$ and $\mathcal{M}_I^2$ are given by

$$m_{R1}^2 \simeq 2cv^2 + \frac{(e-h-2q)^2}{2c-q}w^2\,, \tag{37}$$

$$m_{R2}^2 \simeq qv^2 - \left[\frac{(e-h-2q)^2}{2c-q} - 2d\right] w^2\,, \tag{38}$$

$$m_{I1}^2 = q(v^2 + 4w^2)\,, \tag{39}$$

$$m_{I2}^2 = 0\,, \tag{40}$$

respectively. The masses of the R fields are given up to first order in w^2, whereas the masses of the I fields are exact. The zero eigenvalue corresponds to the linear combination (32). In the GR model without the cubic term in the Higgs potential, we have $t = 0$ (or $q = 0$), and the second eigenvalue of the I fields is zero. This eigenvalue corresponds to the Goldstone boson (majoron) which results from the spontaneous breaking of the U(1) symmetry connected with lepton number conservation. Moreover, in this case m_{R2}^2 is of order w^2, and therefore the Z^0 can decay into the majoron and the light scalar with a decay width of two neutrino flavors [26]. Thus, the GR model is ruled out because of the LEP results. Equations (38) and (39) show that when q is of order one, all physical neutral scalars can be made heavy enough such that the Z^0 cannot decay into them. Consequently, the GR model with a cubic term in the Higgs potential is consistent with the LEP data [27].

For completeness, we also mention the masses of the charged scalars. The mass Lagrangian for the singly charged scalars is given by

$$\mathcal{L}_S^{\pm} = -(H^-, \phi^-)\mathcal{M}_+^2 \begin{pmatrix} H^+ \\ \phi^+ \end{pmatrix} \tag{41}$$

with

$$\mathcal{M}_+^2 = \begin{pmatrix} 2(q+h/2)w^2 & \sqrt{2}v(t^* - v_T h/2) \\ \sqrt{2}v(t - v_T^* h/2) & (q+h/2)v^2 \end{pmatrix}. \tag{42}$$

The field ϕ^+ denotes the charged component of the scalar doublet. One mass eigenvalue of this matrix is zero corresponding to the pseudo-Goldstone boson that gives mass to the W boson. The mass of the single physical scalar with charge +1 is computed as

$$m_+^2 = \left(q + \frac{h}{2}\right)(v^2 + 2w^2)\,. \tag{43}$$

For the mass of the scalar with charge +2, one finds

$$m_{H^{++}}^2 = (h+q)v^2 + 2fw^2\,. \tag{44}$$

Note that, as expected, all physical charged scalars are heavy regardless of whether we set $t = 0$ or not, and hence the Z^0 cannot decay into charged Higgses for $h \sim 1$. Lower bounds on scalar masses from different mechanisms are all below 100 GeV [32] and thus irrelevant for our discussion.

3 Adding a sterile neutrino

Adding a fourth neutrino to the GR model with a cubic term in the Higgs potential, we have to take into account that because of the LEP measurements of the Z^0 decay width, this neutrino must not couple to the Z^0. Thus it has to be a trivial singlet under SU(2)×U(1). Since it has no gauge interactions, it is called a sterile neutrino. In analogy with the fields ℓ_{aR}, we denote it by the right-handed field ν_{sR}. The only new gauge-invariant terms involving the sterile neutrino field are given by

$$\left(-\sum_a h_a \overline{\nu}_{sR} \tilde{\phi}^\dagger L_a + \frac{1}{2} M_s \, \nu_{sR}^T C^{-1} \nu_{sR} \right) + \text{h.c.} \tag{45}$$

The Majorana mass $|M_s|$ of the sterile neutrino is usually assumed to be much larger than the other neutrino masses and could typically be of the order of the GUT scale. Therefore, we opt for introducing a symmetry forbidding the mass term in (45). It turns out, however, that it is not possible to construct such a symmetry, by assigning phase factors to all the multiplets of the model, without forbidding other crucial terms of the model like the cubic term in the Higgs potential. This forces us to introduce a second scalar doublet ϕ_s. Then we can conveniently define a symmetry S by

$$S: \quad \nu_{sR} \to \mathrm{e}^{\mathrm{i}\alpha} \nu_{sR}, \quad \phi_s \to \mathrm{e}^{\mathrm{i}\alpha} \phi_s \,. \tag{46}$$

All other multiplets transform trivially. S forbids the Majorana mass term in (45), provided $\mathrm{e}^{2\mathrm{i}\alpha} \neq 1$. Now instead of (45), we have

$$-\left(\sum_a h_a \overline{\nu}_{sR} \tilde{\phi}_s^\dagger L_a + \text{h.c.} \right) . \tag{47}$$

After spontaneous symmetry breaking, (47) gives the mass term

$$-\left(\frac{v_s}{\sqrt{2}} \sum_a h_a \overline{\nu}_{sR} \nu_{aL} + \text{h.c.} \right) \text{ where } \langle \phi_s \rangle_0 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_s \end{pmatrix} . \tag{48}$$

It is easily seen that the condition $\mathrm{e}^{2\mathrm{i}\alpha} \neq 1$ causes the Higgs potential to be invariant under S (46) interpreted as a continuous symmetry with $\alpha \in \mathbf{R}$. Therefore by spontaneous symmetry breaking, we obtain a Goldstone boson. One can show with the methods of Sect. 2 that the other scalars are heavy and that there is thus no contradiction with the measurement of the Z^0 width. The couplings of the Goldstone boson are similar to those in the model of [33] (see also [23]), which was shown to be compatible with experimental data. The continuous symmetry S allows one to choose $v_s > 0$, and ϕ_s transforms like ϕ (19) under CP. Then the invariance of the term (47) under CP implies $h_a^* = h_a$.

The mass terms (18) and (47) are combined in a 4-neutrino Majorana mass term as

$$\frac{1}{2} \left(\nu_L^T, \nu_{sL}^T \right) C^{-1} \mathcal{M}_{4\nu} \begin{pmatrix} \nu_L \\ \nu_{sL} \end{pmatrix} + \text{h.c.} \tag{49}$$

with

$$\mathcal{M}_{4\nu} = \begin{pmatrix} \mathrm{i}wF & \frac{v_s}{\sqrt{2}} h^T \\ \frac{v_s}{\sqrt{2}} h & 0 \end{pmatrix} , \tag{50}$$

where we have defined the charge-conjugate field $\nu_{sL} \equiv (\nu_{sR})^c$, the 3×3 matrix $F \equiv (f'_{ab})$, and the line vector $h \equiv (h_a)$. Now we fix the notation of the diagonalizing matrices of the mass terms. The mass matrices of the charged leptons (see (17)) and of the neutrinos (see (50)) are diagonalized by

$$W_\ell^\dagger \mathcal{M}_\ell V_\ell = \hat{\mathcal{M}}_\ell \quad \text{and} \quad V_\nu^T \mathcal{M}_{4\nu} V_\nu = \hat{\mathcal{M}}_{4\nu} \,, \tag{51}$$

respectively. From V_ℓ and V_ν, the mixing matrix (1) is computed as

$$U = {V'_\ell}^\dagger V_\nu \quad \text{with} \quad V'_\ell = \begin{pmatrix} V_\ell & 0 \\ 0 & 1 \end{pmatrix} . \tag{52}$$

For the remaining discussion, we will stick to the following order of magnitude assumptions:

$$F \sim h \quad \text{and} \quad v \sim v_s \sim 100\,\text{GeV} \,. \tag{53}$$

This makes our mass matrix (50) analoguous to the one obtained in [14] with the singular see-saw mechanism. With (53) and (3), the elements in the mass matrix (50) are of two different orders of magnitude, represented by the VEVs v_s and $|v_T|$ or $\mu \sim w|f'_{ab}| \ll M \sim v_s|h_a| \;\forall a, b$. With the ordering $m_1 < m_2 < m_3 < m_4$ of the neutrino masses, repeating the arguments of [14], we determine from (50) that

$$m_1,\, m_2 \sim \mu, \quad m_3,\, m_4 \sim M, \quad m_4 - m_3 \sim \mu, \tag{54}$$

and with the definition $\Delta m^2_{jk} = m_j^2 - m_k^2$ we obtain

$$\Delta m^2_{21} \sim \mu^2, \quad \Delta m^2_{43} \sim \mu M, \quad \Delta m^2_{41} \sim M^2 \,. \tag{55}$$

Therefore, in a natural way, three different scales for the mass-squared differences occur. If we set $\Delta m^2_{21} = \Delta m^2_{\text{solar}} \sim 10^{-5}$ eV^2 and $\Delta m^2_{41} = \Delta m^2_{\text{LSND}} \sim 1$ eV^2, we get $\Delta m^2_{43} \sim 3 \times 10^{-3}$ eV^2, which is just the right order of magnitude for Δm^2_{atm}. In this way, we obtain the mass spectrum of Scheme B (2), which forces us to envisage $\nu_e \to \nu_\tau$ MSW transitions as a solution for the solar neutrino deficit and $\nu_\mu \to \nu_s$ transitions for the atmospheric neutrino anomaly. The ratio $\mu/M \sim |v_T|/v_s \sim 3 \times 10^{-3}$ is well below the constraint (29). Note that a solution of the solar neutrino problem by vacuum oscillations with $\Delta m^2_{\text{solar}} \sim 10^{-10}$ eV^2 is not possible in the scenario discussed here.

Finally, we note that with the assumptions (53), the elements of F and h must be very small: if we want $M \sim v_s h$ to be of order 1 eV, then $v_s \sim 100$ GeV implies that F and h must be of order 10^{-11}. However, with all coefficients in F and h being of the same order of magnitude, the structure of the 4-neutrino mass spectrum corresponding to Scheme B is obtained in a natural way, simply by having the two scales given by v_s and $|v_T|$.

4 A discrete symmetry to implement large ν_μ–ν_s mixing

The shortcomings of the model discussed in the previous section and in [14], which were also noticed in [16], are that one still has to resort to fine-tuning in order to specify the large active – sterile neutrino mixing to large ν_μ–ν_s mixing and also to get the correct small ν_e–ν_μ mixing as required by the result of the LSND experiment [10]. In the following, we propose a symmetry called T which replaces the symmetry S of the previous section and removes the first shortcoming. It requires us, however, to enlarge the Higgs content of the scenario in the previous section by an additional scalar doublet. This will allow us to give also a plausible reason for the small ν_e–ν_μ mixing.

In order to implement large ν_μ–ν_s mixing, we require that in the Lagrangian (47), the right-handed neutrino singlet couples to only one left-handed lepton doublet, which we denote by L_3. As we shall see, the nontrivial transformation of the left-handed lepton doublets under T necessitates the introduction of two scalar doublets $\phi_{1,2}$ in the Lagrangian (4) in order to have only nonzero charged-lepton masses. The symmetry T is defined via the prescription

$$\begin{aligned} T: \quad & \nu_{sR} \to \mathrm{i}\nu_{sR}\,, \quad \phi_s \to -\mathrm{i}\phi_s\,, \\ & \phi_2 \to -\phi_2\,, \quad L_3 \to -L_3\,. \end{aligned} \tag{56}$$

All other fields transform trivially under T. Taking into account T, the Yukawa couplings for the two Higgs doublets $\phi_{1,2}$ are given by

$$-\left\{\left(\sum_{a=1}^{3}\sum_{b=1}^{2} c_{ab}\bar{\ell}_{aR}\phi_1^\dagger L_b + \sum_{a=1,2,3} y_a \bar{\ell}_{aR}\phi_2^\dagger L_3\right) + \mathrm{h.c.}\right\}. \tag{57}$$

With the three Higgs doublets $\phi_{1,2,s}$ we have the terms

$$\phi_s^\dagger\phi_1\phi_s^\dagger\phi_2 \quad \text{and} \quad \phi_1^\dagger\phi_2\phi_1^\dagger\phi_2 \tag{58}$$

in the Higgs potential. As a consequence, the only U(1) allowed by the potential is the one associated with the hypercharge. Thus with the symmetry T we forbid a Majorana mass term of the right-handed neutrino singlet and avoid a Goldstone boson at the same time.

Defining $\langle\phi_k^0\rangle = v_k/\sqrt{2}$ $(k=1,2,s)$, we assume that all doublet VEVs are of the same order of magnitude. Now, with the two cubic terms pertaining to $\phi_{1,2}$ and the quartic terms (58) in the Higgs potential, CP can be broken explicitly or spontaneously in the Higgs sector. In the following, we will stick to CP conservation and assume real VEVs for simplicity. The Yukawa couplings (57) give the mass matrix for the charged leptons:

$$\mathcal{M}_\ell = \left(\frac{v_1}{\sqrt{2}}\begin{pmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \\ c_{31} & c_{32}\end{pmatrix}, \frac{v_2}{\sqrt{2}}\begin{pmatrix} y_1 \\ y_2 \\ y_3\end{pmatrix}\right). \tag{59}$$

From this equation, it is obvious that a third scalar doublet ϕ_2 is needed to reproduce the charged-lepton mass spectrum. Because of the symmetry T, the neutrino mass matrix splits into two 2×2 matrices:

$$\mathcal{M}_{4\nu} = \begin{pmatrix} \mathcal{M}_{12} & 0 \\ 0 & \mathcal{M}_{3s}\end{pmatrix}, \tag{60}$$

with

$$\mathcal{M}_{12} = \mathrm{i}w\begin{pmatrix} f'_{11} & f'_{12} \\ f'_{12} & f'_{22}\end{pmatrix} \quad \text{and} \quad \mathcal{M}_{3s} = \begin{pmatrix} iwf'_{33} & \frac{v_s}{\sqrt{2}}h_3 \\ \frac{v_s}{\sqrt{2}}h_3 & 0\end{pmatrix}. \tag{61}$$

Let us consider the matrix $\mathcal{M}_{3s}$. Up to order w, it gives the neutrino masses

$$\frac{1}{\sqrt{2}}|v_s h_3| \pm \frac{1}{2}wf'_{33} \tag{62}$$

and a mixing angle θ_{3s} obtained by

$$\sin^2 2\theta_{3s} \simeq 1 - \frac{1}{2}\left(\frac{wf'_{33}}{v_s h_3}\right)^2. \tag{63}$$

With $v_s \sim v_{1,2}$, $f'_{ab} \sim h_3$ (53) and (29), $\sin^2 2\theta_{3s}$ is 1 for all practical purposes, and naturally we want to associate the matrix $\mathcal{M}_{3s}$ with the $\nu_\mu - \nu_s$ solution of the atmospheric neutrino problem. Furthermore, the other 2×2 mass matrix $\mathcal{M}_{12}$ has all matrix elements of the same order of magnitude and therefore suggests that the solar neutrino problem be explained by $\nu_e - \nu_\tau$ oscillations with the large-angle MSW solution.

The diagonalization matrix V_ν of the neutrino mass matrix (60) consists of two 2×2 submatrices, i.e.,

$$V_\nu = \begin{pmatrix} V_{12} & 0 \\ 0 & V_{3s}\end{pmatrix}. \tag{64}$$

So V_ν does not have, e.g., ν_e–ν_μ mixing necessary to describe the LSND experiment, provided we associate the submatrices of $\mathcal{M}_\nu$ (60) with neutrino flavors, as was done in the previous paragraph. However, in order to obtain the mixing matrix U, we have to multiply V_ν with $V_\ell'^\dagger$ (see (1) and (52)), which is determined by the diagonalization of $\mathcal{M}_\ell$ (17). Our model does not specify $\mathcal{M}_\ell$. In order to proceed further, we make the following assumption regarding V_ℓ: In analogy with the quark sector we assume that V_ℓ is close to a diagonal phase matrix. This amounts to $|(V_\ell)_{1e}| \simeq |(V_\ell)_{2\tau}| \simeq |(V_\ell)_{3\mu}| \simeq 1$, since these elements correspond to the diagonal elements of V_ℓ in our model. All other elements are assumed to be small.

Clearly, this assumption is in agreement with the scenarios for the atmospheric and solar neutrinos proposed above. Let us now discuss how the result of the LSND experiment fits into the model. This experiment measures the short-baseline transition amplitude

$$P^{(\mathrm{SBL})}_{\bar\nu_\mu\to\bar\nu_e} = A_{e;\mu}\sin^2\frac{\Delta m^2_{41}L}{4E_\nu}, \tag{65}$$

where L is the distance between the neutrino source and detector, E_ν is the neutrino energy, and the oscillation amplitude $A_{e;\mu}$ is obtained from the mixing matrix as

$$A_{e;\mu} = 4\left|\sum_{j=3,4} U^*_{ej}U_{\mu j}\right|^2. \tag{66}$$

Considering the structure (64) of V_ν, one finds

$$A_{e;\mu} = 4\,|(V_\ell)_{3e}|^2|(V_\ell)_{3\mu}|^2 \simeq 4\,|(V_\ell)_{3e}|^2\,. \qquad (67)$$

In the last step, we have used our assumption about V_ℓ. The experimental result of the LSND experiment, taking into account other short-baseline experiments which have seen no indication in favor of neutrino oscillations, is expressed as [10],

$$2\times 10^{-3} \lesssim A_{e;\mu} \lesssim 3\times 10^{-2}\,, \qquad (68)$$

where the bounds result from the LSND-allowed region (90% CL). Thus, from (66) and (67), it follows that $|(V_\ell)_{3e}|$ is of the order of 10^{-2} to 10^{-1}, conforming with the above assumption as expected.

We conclude this section with some remarks about the scalars. Now there are two cubic terms corresponding to $\phi_{1,2}$, and therefore two coupling constants $t_{1,2}$ in the potential (see (10)) which must both be much smaller than the doublet VEVs and of the order of the triplet VEV. The assumption of CP conservation simplifies the discussion of the neutral scalar masses because it causes the 8×8 scalar mass matrix to split into two 4×4 mass matrices, one for the R fields and one for the I fields (see Sect. 1). One can again show that all physcial neutral scalars are heavy of the order of the doublet VEVs. The same is true for the charged scalars.

5 Conclusions

In this paper we have constructed a 4-neutrino model based on the Gelmini – Roncadelli model, which extends the standard model by a scalar triplet Δ leading to Majorana neutrino masses at the tree level. In order to prevent the Z^0 decay into light neutral scalars, we have explicitly broken the lepton number of the original GR model by a cubic term in the Higgs potential. We have introduced a sterile neutrino and coupled it to the standard lepton gauge doublets by a separate Higgs doublet ϕ_s. It is well known that the triplet VEV must be much smaller than the doublet VEVs because of the tree-level relation $M_W = M_Z \cos\theta_W$. One of the main points of our model is to exploit the presence of the two scales represented by the triplet and doublet VEVs. In this way, assuming that the Δ and ϕ_s couplings are of the same order of magnitude, we immediately arrive at a model which reproduces the neutrino mass spectrum of Scheme B (2), one of the two schemes allowed by all present neutrino oscillation data. This model, described in Sect. 3, is completely analogous to the model of [14], which invokes the singular see-saw mechanism. However, in this case, the heavy scale of the see-saw mechanism is quite low, on the order of keV. Our model avoids this – as we have only four light neutrinos – at the expense of the triplet VEV being much smaller than the doublet VEVs occurring in the model. Of course, the smallness of the triplet VEV can be obtained only by fine-tuning in the Higgs potential, and the hope is that in a more complete theory, this problem of fine-tuning is resolved. Note that in our model, neutrinoless double β decay is suppressed, because in Scheme B the effective Majorana mass $\langle m\rangle = \sum_j U_{ej}^2 m_j$ is small [6], and mechanisms involving the Higgs sector are negligible due to the tiny scalar–fermion couplings.

The scenario of Sect. 3 automatically leads to a large active–sterile neutrino mixing. However, any linear combination of the active neutrinos could have this large mixing. In Sect. 4 we introduced a symmetry which splits the 4×4 Majorana neutrino mass matrix into two 2×2 matrices. The diagonalization matrices of both 2×2 matrices contain a large angle, one of them is $\pi/4$ for all practical purposes. In this version of the model, we need three Higgs doublets. Neglecting for a moment the part of the mixing matrix U coming from the charged-lepton sector (see (52)), the mixing matrix also separates into two 2×2 matrices. In this way, we naturally obtain a model where the solar neutrino problem is explained by large mixing angle MSW $\nu_e \to \nu_\tau$ transitions and the atmospheric neutrino problem by $\nu_\mu \to \nu_s$ transitions with mixing angle $\pi/4$. With the assumption that in the charged-lepton sector, the left-handed diagonalization matrix of the mass matrix is close to a diagonal phase matrix, the scenario just described is not very much disturbed. Moreover, one can exploit V_ℓ (51) to incorporate the LSND result of small ν_e–ν_μ mixing, which is forbidden if V_ℓ is diagonal.

This assumption about the charged-lepton sector is certainly a weak point of our model, but in any case, we have no explanation for the charged-lepton spectrum either. Furthermore, the assumption of an equal order of magnitude of the Δ and ϕ_s couplings leads to very small coupling constants of order 10^{-11} to obtain the smallness of the neutrino masses relative to M_W and M_Z. In addition, this has to find a natural explanation in a larger theory. Despite these shortcomings, we want to stress that our model requires only the minimal extension of the fermionic sector of the standard model necessary for a 4-neutrino scheme and that looking for an explanation of the 4-neutrino mass spectrum indicated by the experimental data in terms of VEVs of scalar multiplets could provide interesting clues for theories with scales beyond the gauge boson masses of the standard model.

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